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Similitude in Hydraulic Modelling

Dr Valentin Heller

Environmental Fluid Mechanics and Geoprocesses Research Group

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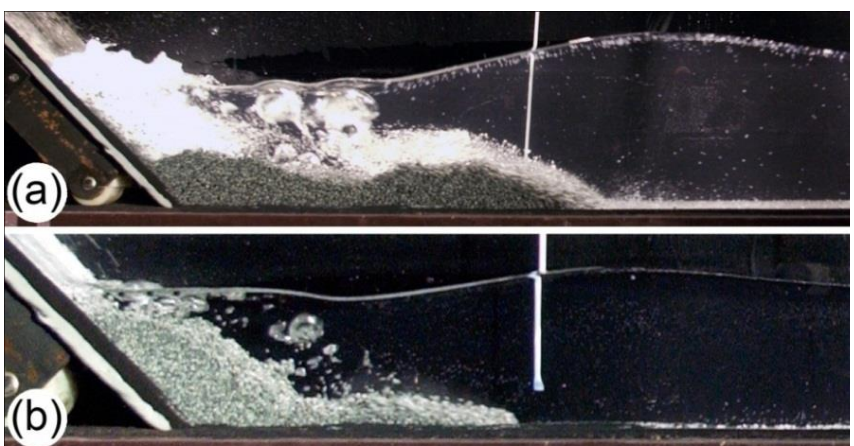

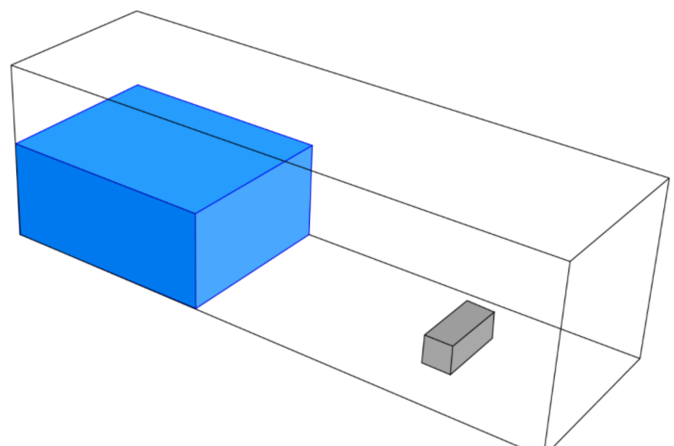

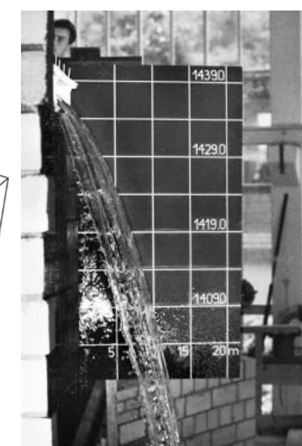

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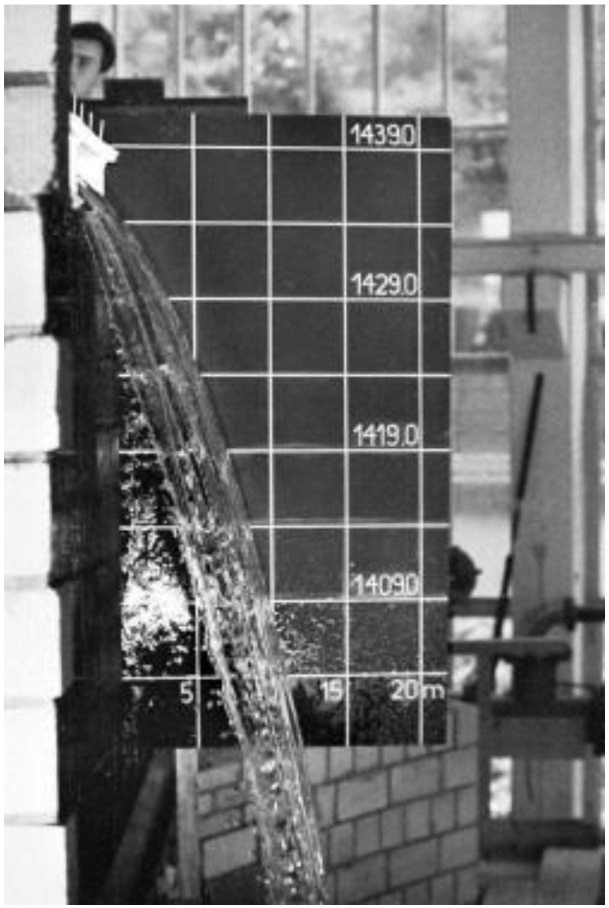
Background Hydraulic Modelling

Clockwise from to left: Ski jump at Karakaya hydropower dam, sediment transport, tidal turbine (SeaGen), weir, flooding, breaking wave, floating wave energy converter (Plemais)

Similarity: Gebidem Dam, Switzerland

Laboratory model at 1:30



Full scale (prototype)

Jet trajectory ✔

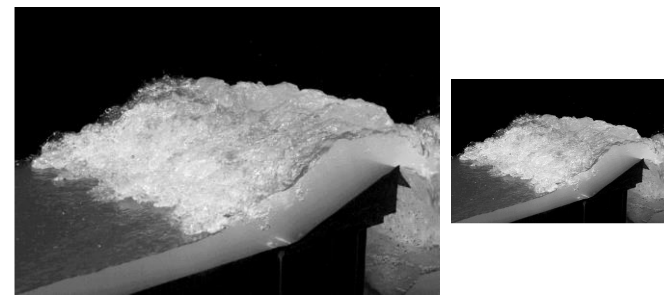
Air concentration ✖

Scale factor $\lambda = L_P/L_M$ with L_P = characteristic length in the **P**rototype and L_M = corresponding length in the **M**odel

Froude Scaling Laws: Mechanical (perfect) similarity

A physical scale model satisfying *mechanical similarity* is completely similar to its prototype and involves no scale effects (Hughes 1993).

Mechanical similarity requires three criteria to be satisfied:



(i) **Geometric similarity**:

Similarity in shape, i.e. all length dimensions in the model are λ times shorter than of its prototype ($\lambda = L_P/L_M$)

(ii) **Kinematic similarity**:

Geometric similarity and similarity of motion between model and prototype particles

(iii) **Dynamic similarity**:

Requires geometric and kinematic similarity and in addition that all force ratios in the two systems are identical

Froude Scaling Laws: Mechanical (perfect) similarity

The **most relevant forces** in fluid dynamics are:

- Inertial force
- Gravitational force
- Viscous force
- Surface tension force
- Elastic compression force
- Pressure force

The relevant **force ratios** are:

- Froude number F = (inertial force/gravity force)$^{1/2}$
- Reynolds number R = inertial force/viscous force
- Weber number W = inertial force/surface tension force
- Cauchy number C = inertial force/elastic force
- Euler number E = pressure force/inertial force

Challenge: Only the most relevant force ratio can be identical between a model and its prototype, if the same fluid is used, and mechanical similarity is impossible. The most relevant force ratio is selected and the remaining result in **scale effects**.

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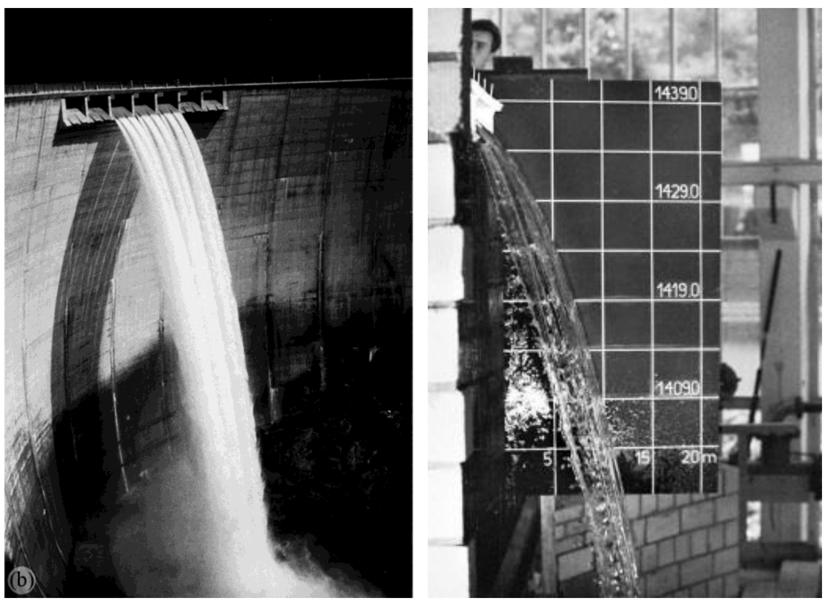
Froude Scaling Laws: Froude similarity

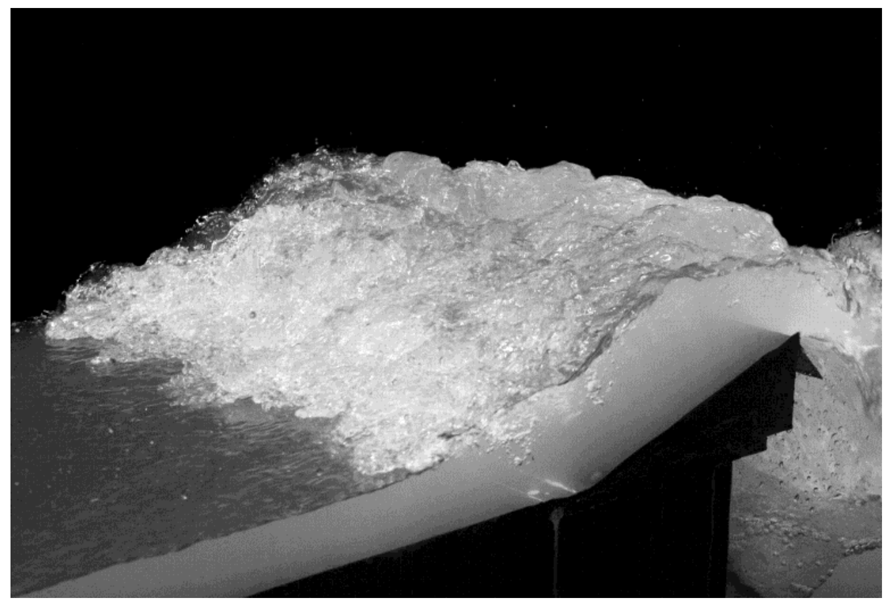
Froude similarity $F_M = F_P$ (basic assumption)

For phenomena where gravity and inertial forces are dominant and effect of remaining forces such as kinematic viscosity are small.

Most water phenomena are modeled after Froude, particularly free surface flows (hydraulic structures, waves, wave energy converters, fluvial hydraulics including sediment transport, etc.)



Gebidem dam



Hydraulic jump modelled after Froude

Froude scaling laws

Parameter	Dimension	Froude
	Geometric similarity	
Length	[L]	λ
Area	$[L^2]$	λ^2
Volume	$[L^3]$	λ^3
Rotation	[-]	1
	Kinematic similarity	
Time	[T]	$\lambda^{1/2}$
Velocity	$[LT^{-1}]$	$\lambda^{1/2}$
Acceleration	$[LT^{-2}]$	1
Discharge	$[L^3T^{-1}]$	$\lambda^{5/2}$
	Dynamic similarity	
Mass	[M]	λ^3
Force	$[MLT^{-2}]$	λ^3
Pressure and stress	$[ML^{-1}T^{-2}]$	λ
Energy and work	$[ML^2L^{-2}]$	λ^4
Power	$[ML^2T^{-3}]$	$\lambda^{7/2}$

Example with discharge rate Q:

Given:
$Q_M = 0.01\ m^3/s$ in a $1:\lambda = 1:20$ scale model

Unknown:
discharge in the prototype Q_P

Solution:
$Q_P = \lambda^{5/2} Q_M = 20^{5/2} 0.01 = 17.89\ m^3/s$

Why do scale effects matter?

Failure of Sines breakwater in 1978/9

Strong enough in the scale model investigation (one reason for the failure were scale effects due to the incorrect scaling of the structural properties)

Images of destroyed Sines breakwater, 1978/9

Illustration scale effects: Landslide-tsunamis

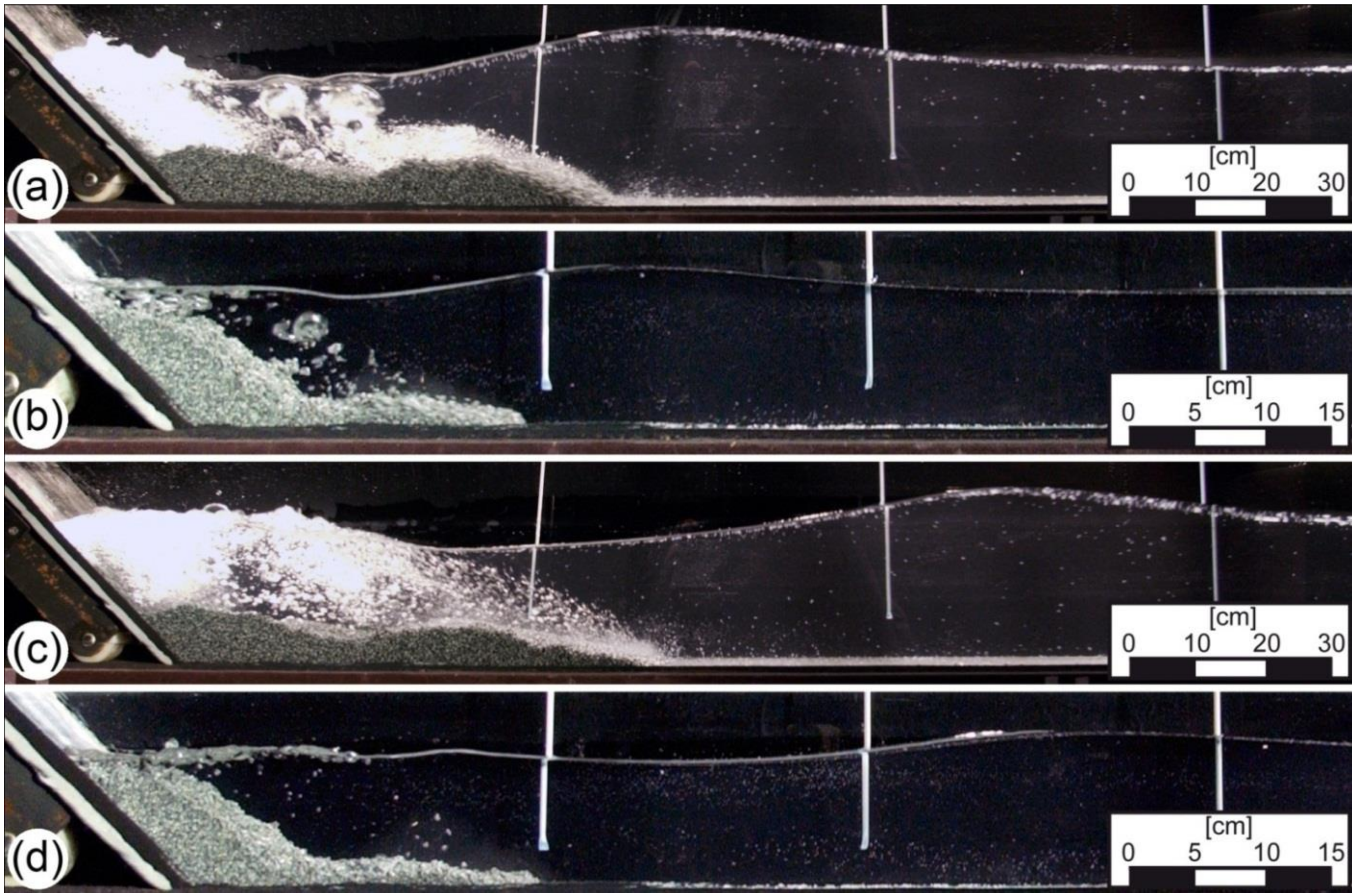
Subaerial landslide-tsunami generation at 1:2 and 1:4 (Heller et al., 2008)

1:2

1:4

1:2

1:4



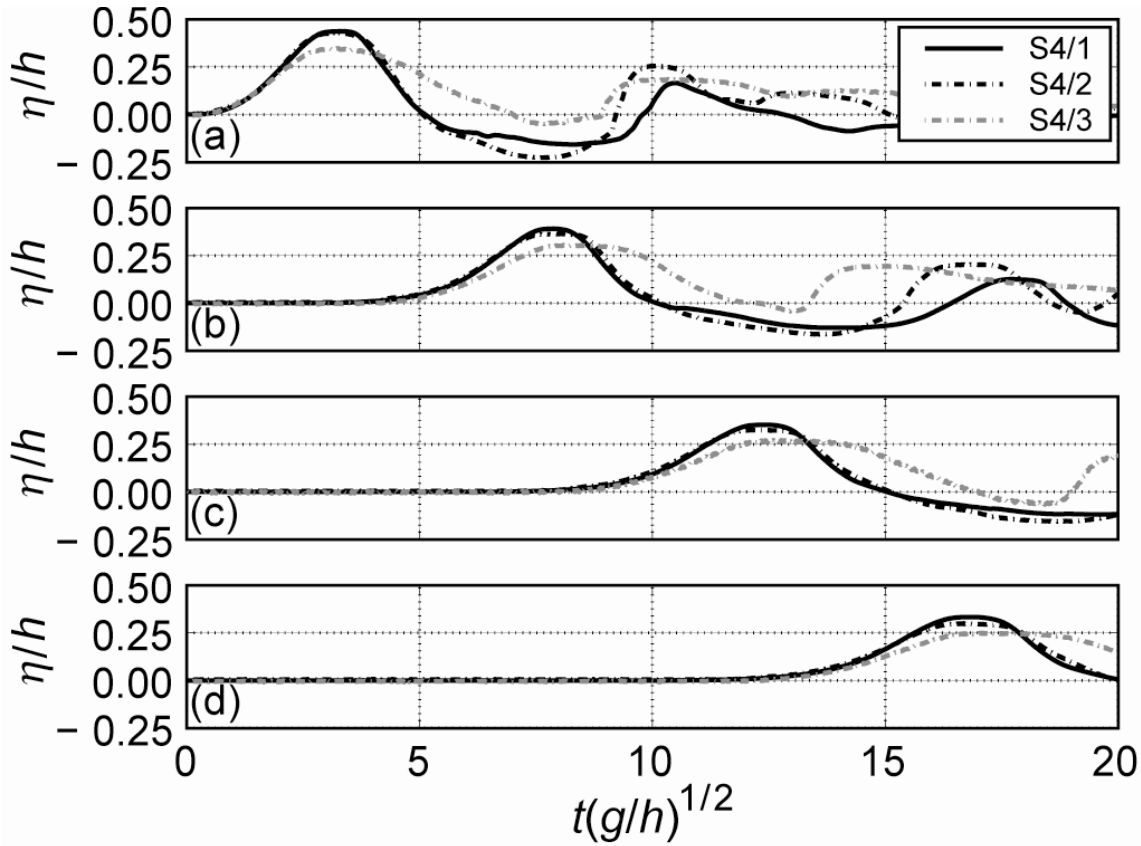
F = 2.5
W = 5350
R = 290000

F = 2.5
W = 1340
R = 103400

F = Froude number; W = Weber number; R = Reynolds number

Illustration scale effects: Landslide-tsunamis

Wave profiles of subaerial landslide-tsunamis at 1:1, 1:2 and 1:4



S4/1: h = 0.400 m (1:1)
S4/2: h = 0.200 m (1:2)
S4/3: h = 0.100 m (1:4)

Scale effects relative to the maximum wave amplitude are negligible (<2%) if:

Reynolds number: R ≥ 300000 Weber number: W ≥ 5000

Avoidance: With rules of thumb

Satisfy limiting criteria (limiting size of the experiment)

In Froude models: $\mathrm{R} = LV/\nu > \mathrm{R}_{limit}$, $\mathrm{W} = \rho V^2 L/\sigma > \mathrm{W}_{limit}$ etc.

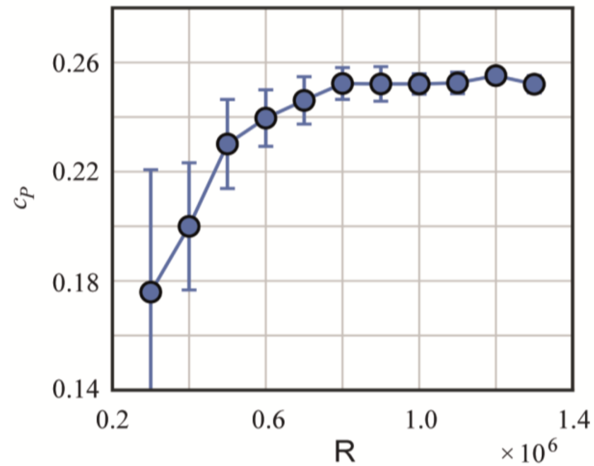
In practice rules of thumb are often applied. Some examples:

- Linear wave propagation is affected less than 1% by surface tension if the wave period > 0.35 s (corresponding to a wave lenght > 0.17 m, Hughes 1993)
- Free surface water flows should be > 5 cm to avoid significant surface tension scale effects (e.g. Heller 2011)
- Wave height to measure wave force on slope during wave breaking should be larger than 0.50 m (Skladnev and Popov 1969)
- Free surface air-water flows should be conducted at $\mathrm{W}^{0.5} > 140$ (Pfister and Chanson 2012)

Avoidance: R invariance

For Froude models when **both** F and R are a priori **relevant**

Horizontal axis tidal turbine

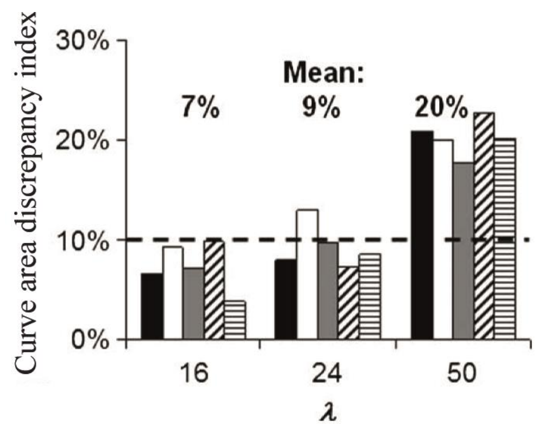


Reynolds number $R = LV/\nu$

L = Length
V = Velocity
ν = kinematic fluid viscosity

Asymptotically approached power coefficient c_P level with R for a tidal energy converter suggesting limiting R = 800,000 (Bachant and Wosnik 2016)

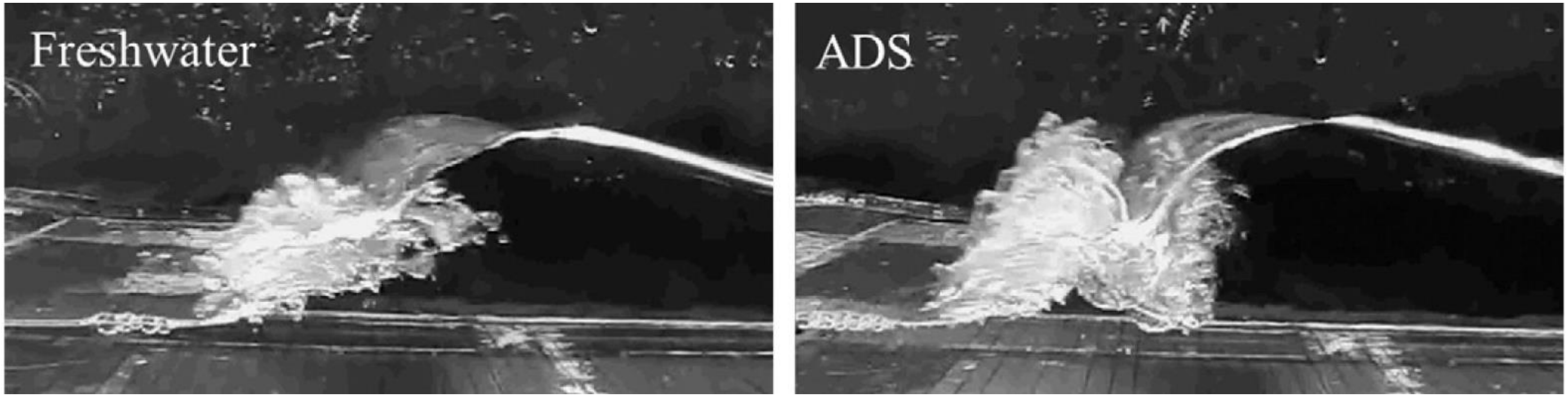
Solute transport in a chlorine contact tank



Variation of curve area discrepancy index with scale and discharge requiring a certain model size $\lambda \leq 24$ corresponding to a limiting R (Teixeira and Rauen 2014)

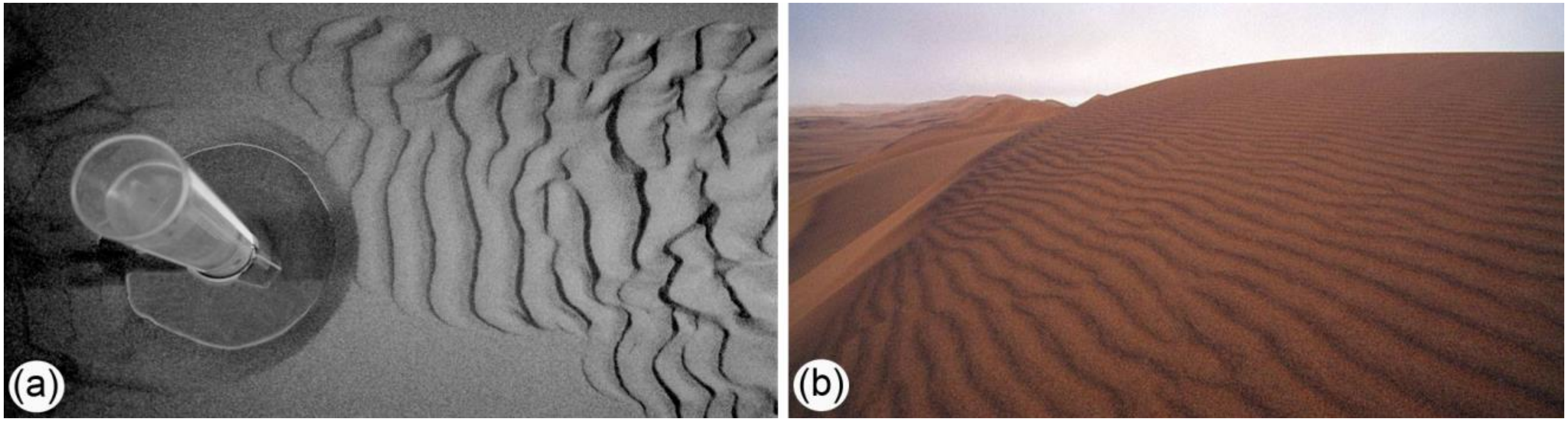
Avoidance: Replacement of fluid

Water replaced by water-isopropyl alcohol (reduced surface tension, increased W)



Stagonas et al. (2011)

Water replaced by air (reduced kinematic viscosity, increased R)



Heller (2011)

Compensation

Compensation is achieved by distorting a model geometry by giving up exact geometric similarity of some parameters in favour of an improved model-prototype similarity.

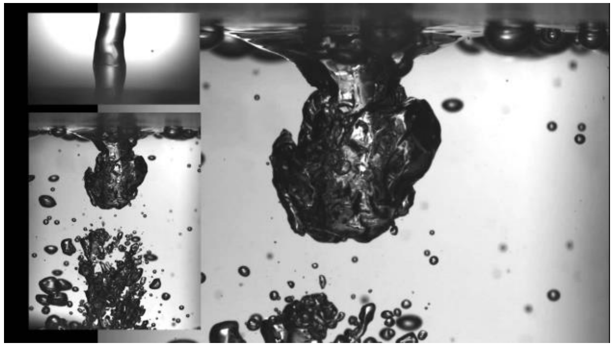
Examples:

- Distorted models: the length λ_L scale factor of a model (typically of a river) is smaller than the height and width scale factor λ to compensate increased viscose and surface tension effects with a larger flow velocity
- The grain diameter d_g in sediment transport is often not scaled with the same scale factor λ as the model main dimensions to avoid $d_g < 0.5$ mm for which cohesion becomes relevant. A larger grain diameter is selected and the grain density is reduced to compensate for this


Distorted river model


Sand hold together by cohesion

Work of PhD student Daniele Catucci



- He derived novel scaling laws (NSL) analytically from the governing equations (including surface tension) with the **Lie group transformation**
- He proves then numerically with OpenFOAM that these ideal scaling laws involve **no scale effects**

Continuity Equation

$$\frac{\partial U_j}{\partial x_j} = 0$$

Reynolds-averaged Navier-Stokes Equation

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\nu \frac{\partial U_i}{\partial x_j} - \overline{u_i u_j}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + g_i + \boxed{\frac{1}{\rho} f_\sigma}$$

Surface tension

One parameter Lie-group transformation

$$\phi = \beta^{\alpha_\phi} \bar{\phi}$$

α_ϕ = Scaling exponent
β = Scaling parameter
ϕ = Variable in the original space
$\bar{\phi}$ = Variable in the scaled space

Work of PhD student Daniele Catucci

variables	scaling conditions in terms of α_x, α_t and α_ρ (novel scaling laws)		scaling conditions in terms of α_x, $\alpha_\rho = \alpha_g = 0$ (precise Froude scaling laws)	
	exponents	scaling ratios	exponents	scaling ratios
length (m)	α_x	β^{α_x}	α_x	$\beta^{\alpha_x} = \lambda$
time (s)	α_t	β^{α_t}	$\alpha_t = 0.5\alpha_x$	$\beta^{0.5\alpha_x} = \lambda^{0.5}$
density (kg m^{-3})	α_ρ	β^{α_ρ}	$\alpha_\rho = 0$	$\beta^0 = 1$
velocity (m s^{-1})	$\alpha_U = \alpha_x - \alpha_t$	$\beta^{\alpha_x - \alpha_t}$	$\alpha_U = 0.5\alpha_x$	$\beta^{0.5\alpha_x} = \lambda^{0.5}$
pressure (Pa)	$\alpha_p = 2\alpha_x - 2\alpha_t + \alpha_\rho$	$\beta^{2\alpha_x - 2\alpha_t + \alpha_\rho}$	$\alpha_p = \alpha_x$	$\beta^{\alpha_x} = \lambda$
gravitational acceleration (m s^{-2})	$\alpha_g = \alpha_x - 2\alpha_t$	$\beta^{\alpha_x - 2\alpha_t}$	$\alpha_g = 0$	$\beta^0 = 1$
viscosity (m^2 s^{-1})	$\alpha_\nu = 2\alpha_x - \alpha_t$	$\beta^{2\alpha_x - \alpha_t}$	$\alpha_\nu = 1.5\alpha_x$	$\beta^{1.5\alpha_x} = \lambda^{1.5}$
surface tension (N m^{-1})	$\alpha_\sigma = 3\alpha_x - 2\alpha_t + \alpha_\rho$	$\beta^{3\alpha_x - 2\alpha_t + \alpha_\rho}$	$\alpha_\sigma = 2\alpha_x$	$\beta^{2\alpha_x} = \lambda^2$
curvature of the free surface (1 m^{-1})	$\alpha_\kappa = \alpha_x^{-1}$	$\beta^{\alpha_x^{-1}}$	$\alpha_\kappa = \alpha_x^{-1}$	$\beta^{\alpha^{-1}} = \lambda^{-1}$
eddy viscosity (m^2 s^{-1})	$\alpha_{\nu_t} = 2\alpha_x - \alpha_t$	$\beta^{2\alpha_x - \alpha_t}$	$\alpha_{\nu_t} = 1.5\alpha_x$	$\beta^{1.5\alpha_x} = \lambda^{1.5}$
Reynolds stresses (m^2 s^{-2})	$\alpha_{\langle u_i, u_j \rangle} = 2\alpha_x - 2\alpha_t$	$\beta^{2\alpha_x - 2\alpha_t}$	$\alpha_{\langle u_i, u_j \rangle} = \alpha_x$	$\beta^{\alpha_x} = \lambda$
turbulent kinetic energy (m^2 s^{-2})	$\alpha_k = 2\alpha_x - 2\alpha_t$	$\beta^{2\alpha_x - 2\alpha_t}$	$\alpha_k = \alpha_x$	$\beta^{\alpha_x} = \lambda$
dissipation (m^2 s^{-3})	$\alpha_\epsilon = 2\alpha_x - 3\alpha_t$	$\beta^{2\alpha_x - 3\alpha_t}$	$\alpha_\epsilon = 0.5\alpha_x$	$\beta^{0.5\alpha_x} = \lambda^{0.5}$
production of turbulence due to horizontal velocity gradients (m^2 s^{-3})	$\alpha_{P_k} = 2\alpha_x - 3\alpha_t$	$\beta^{2\alpha_x - 3\alpha_t}$	$\alpha_{P_k} = 0.5\alpha_x$	$\beta^{0.5\alpha_x} = \lambda^{0.5}$

Catucci et al. (2021)

- The Froude scaling laws are a special case of the novel scaling laws (NSLs)
- The NSLs are more universal and flexible than the Froude scaling laws

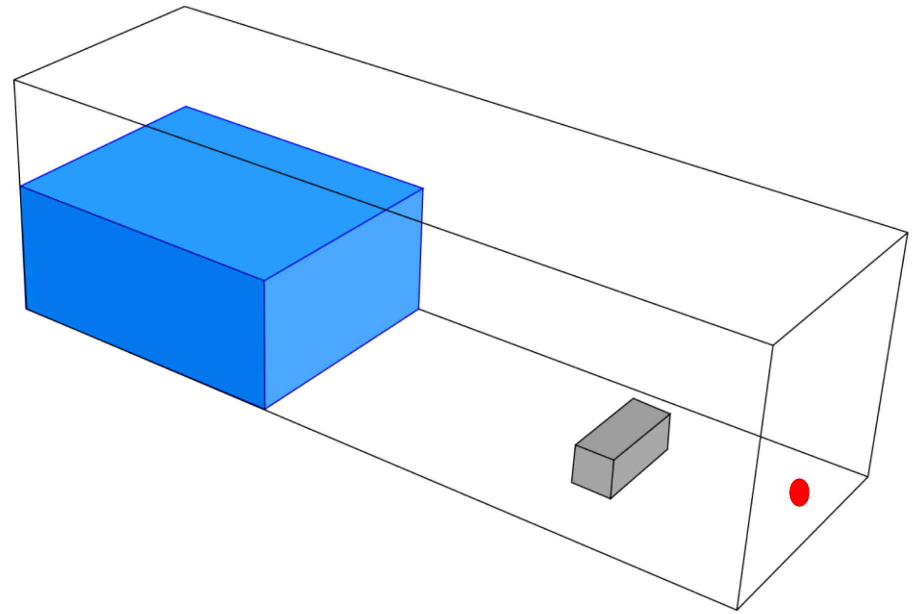
Validation NSLs with dam break wave in OpenFOAM

Past case

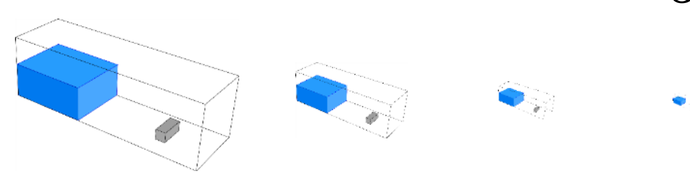
St. Francis Dam collapse in 1928

Scale series to quantify scale effects

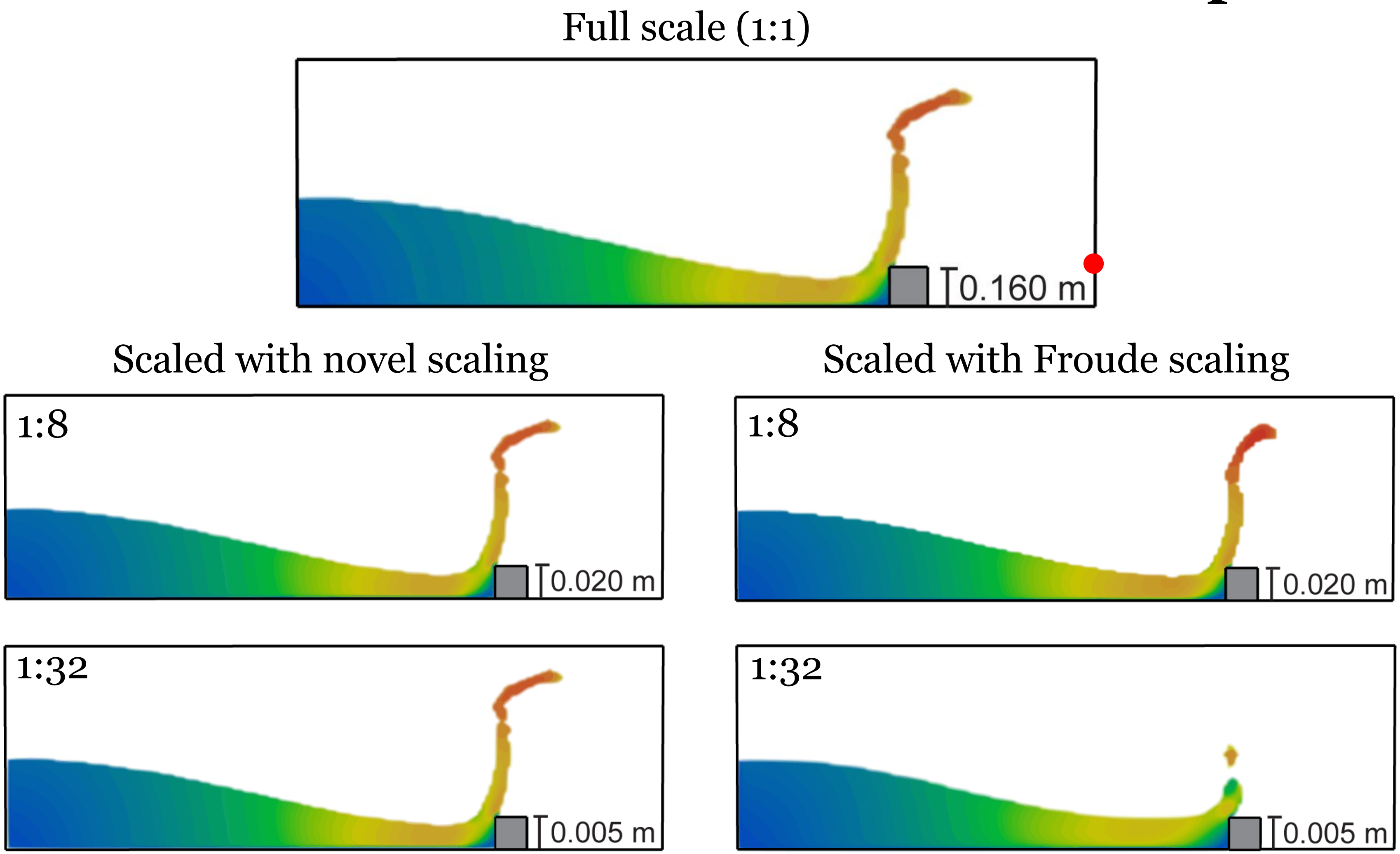
1:1



1:4 1:8 1:16 1:32



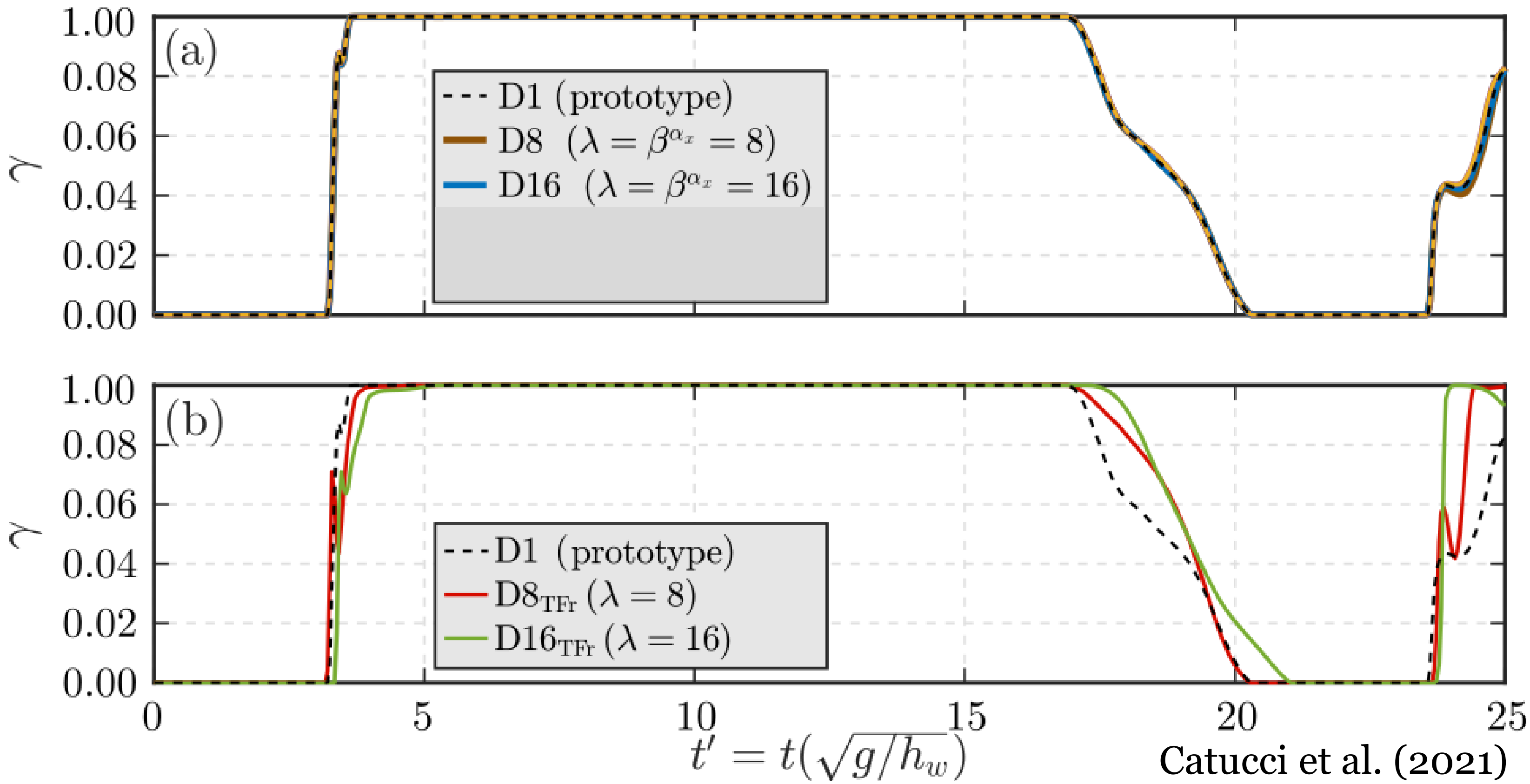
Validation NSLs with dam break wave in OpenFOAM



- The novel scaling laws involve no scale effects in contrast to Froude scaling

Validation NSLs with dam break wave in OpenFOAM

Investigation of Weber number effects with phase fraction γ

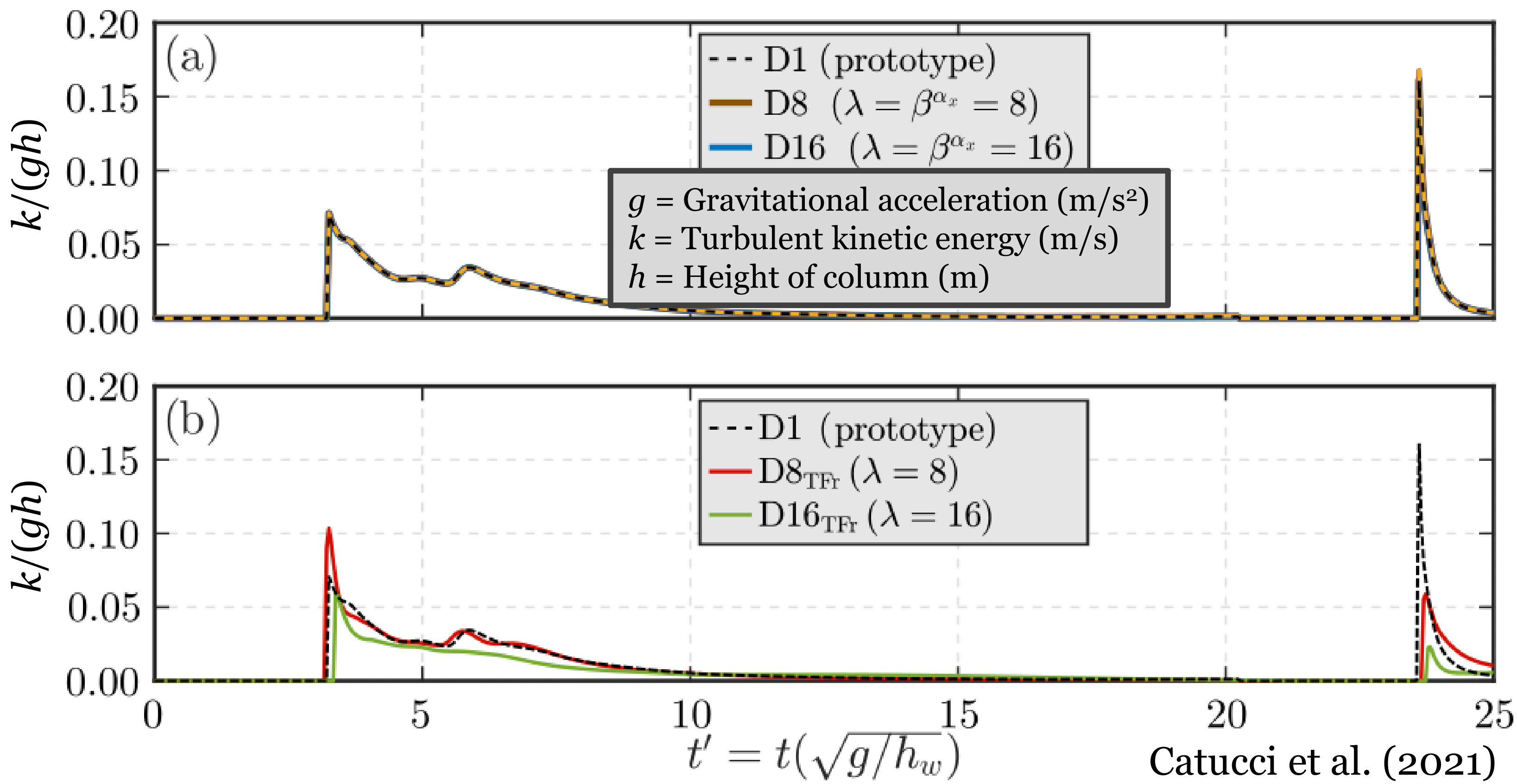


Catucci et al. (2021)

- The curves based on the **novel scaling laws** collapse (**no scale effects**); the ones based on **Froude scaling** result **in scale effects**

Validation NSLs with dam break wave in OpenFOAM

Investigation of Reynolds number effects with turbulent kinetic energy



Catucci et al. (2021)

- The curves based on the **novel scaling laws** collapse (**no scale effects**); the ones based on **Froude scaling** result **in scale effects**

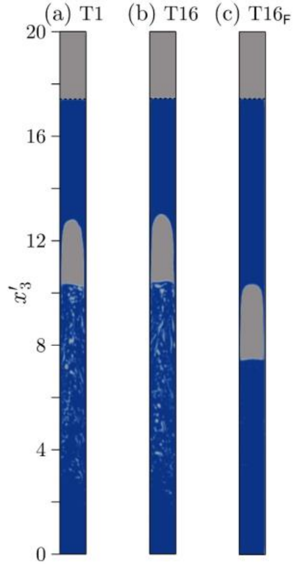


Outlook

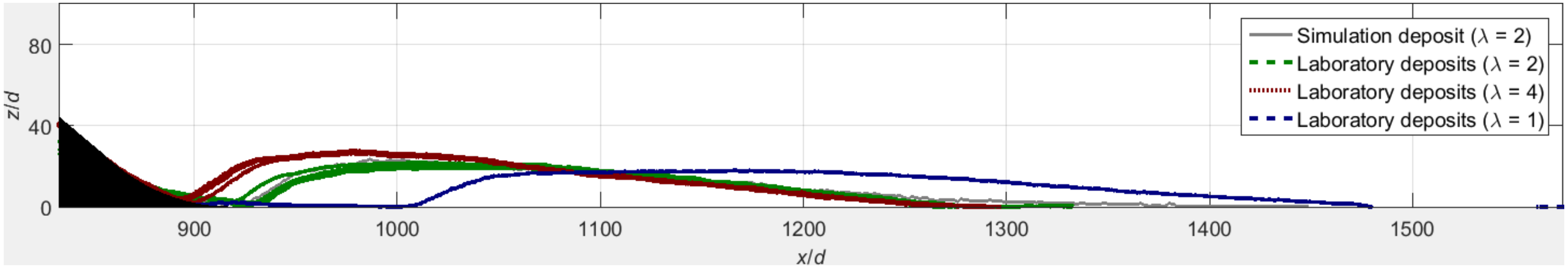
- Ganular slides (Drs Sazeda Begam & Matthew Kesseler)
- Validate NSLs in the lab and NSLs including air compressibility

Bingham Canyon Mine landslide

Plunging jet in the lab



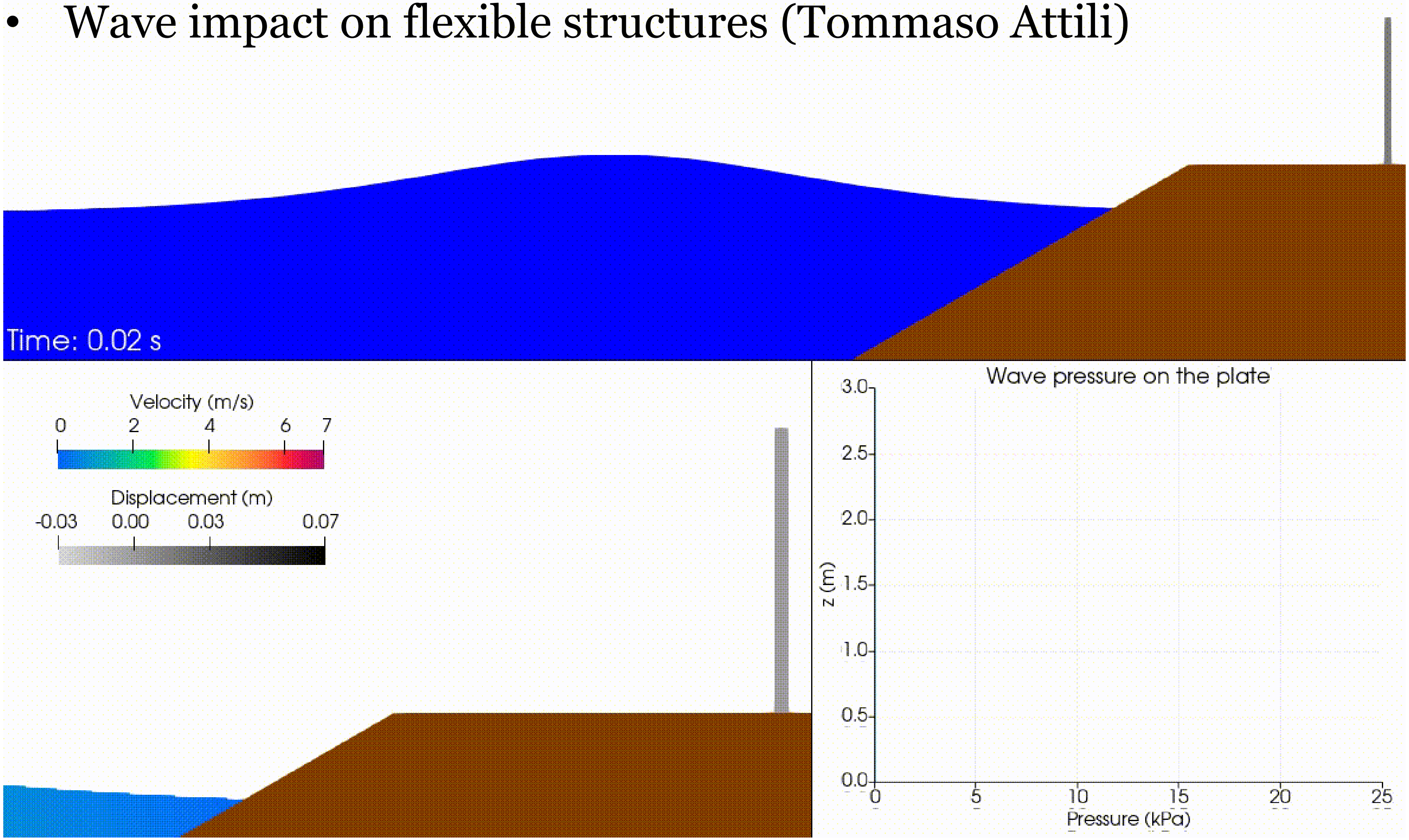
Taylor bubbles



Side view of slide deposits from experiments at 1:1, 1:2 and 1:4 (Kesseler et al., 2020)

Outlook

- Wave impact on flexible structures (Tommaso Attili)



Attili et al. (2022)

Conclusions

- The basics of **Froude similarity**, applied for nearly 100 years in hydraulic modelling, has been covered
- A major limitation of Froude scaling laws are **scale effects**, which become more dominant with decrease model size
- Strategies to deal with significant scale effects have been introduced, including **avoidance and compensation**
- **Novel scaling laws**, which are more flexible and universal that the Froude scaling laws, have been derived to model **air-water flows**
- **These novel scaling laws** work theoretically and were validated with numerical experiments, but they still need to be validated within laboratory experiments
- A range of **ongoing research projects** to understand and achieve similitude in hydraulic modelling have been shown

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Some key references

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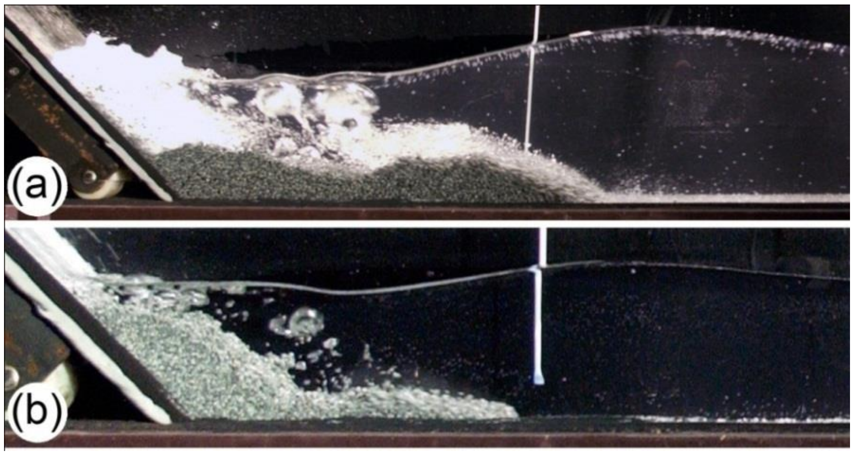

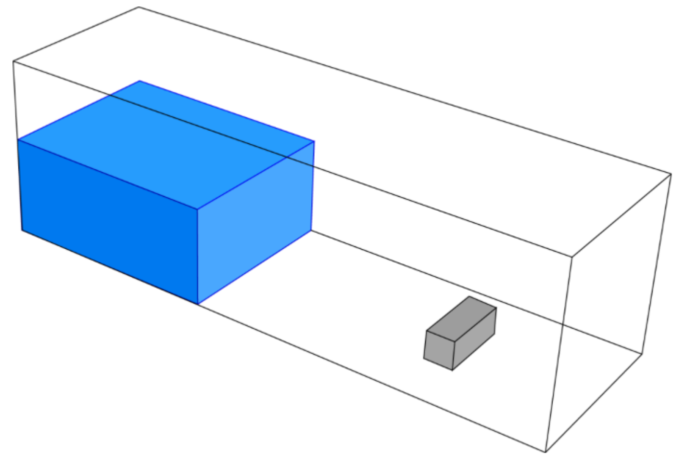

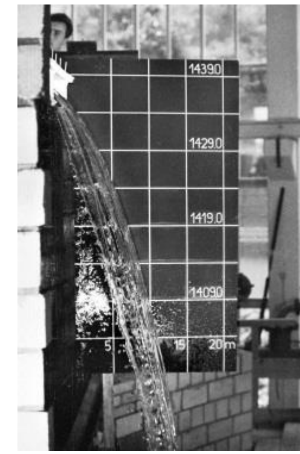

Dr Valentin Heller
www.drvalentinheller.com

Environmental Fluid Mechanics and Geoprocesses Research Group

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