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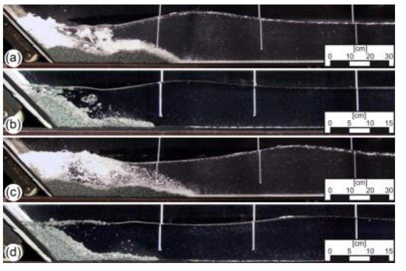
Coastlab

Model-Prototype Similarity



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4th CoastLab Teaching School, Wave and Tidal Energy, Porto, 17-20th January 2012

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Content

- Introduction
- Similarities
- Scale effects
- Reaching model-prototype similarity
- Dealing with scale effects
- Conclusions

Lecture follows Heller (2011)

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Introduction

Why do quantities between model and prototype disagree?

Measurement effects: due to non-identical measurement techniques used for data sampling in the model and prototype (intruding versus non-intruding measurement system etc.).

Model effects: due to the incorrect reproduction of prototype features such as geometry (2D modelling, reflections from boundaries), flow or wave generation techniques (turbulence intensity level in approach flow, linear wave approximation) or fluid properties (fresh instead of sea water).

Scale effects: due to the inability to keep each relevant force ratio constant between the scale model and its real-world prototype.

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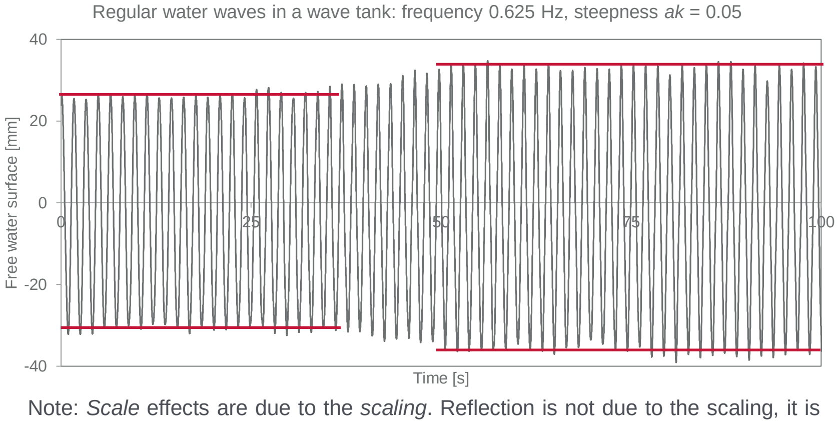
Introduction

Example model effects

Reflections from beach or from non-absorbing wave maker (without WEC)

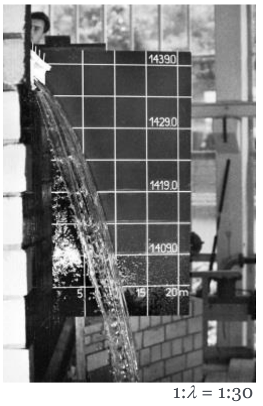


Note: *Scale* effects are due to the *scaling*. Reflection is not due to the scaling, it is therefore not a scale but a model effect.

Introduction

Example scale effects

Miniature universe

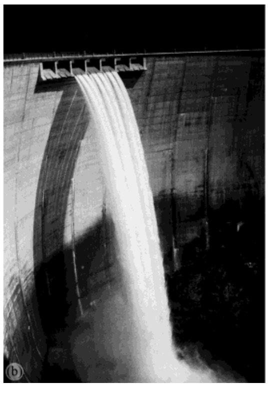


1:λ = 1:30

Jet trajectory ✔

Air concentration ✖

Real-world prototype



Scale ratio or scale factor $\lambda = L_P/L_M$ with L_P = a characteristic length in the real-world **p**rototype and L_M = corresponding length in the **m**odel

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Introduction

Why are scale effects relevant?

- Scale model test results may be *misleading* if up-scaled to full scale (e.g. incorrect economic prediction of a WEC based on power measurements)
- Scale effects may be responsible for *failures* (Sines breakwater, river section is unable to deal with predicted discharge)
- Whether or not scale effects are significant depends on the *relative importance of the involved forces*. Understanding which forces are *relevant* and which can be *neglected* is of key importance in physical, numerical and mathematical modelling:
  Does a numerical simulation require an additional term to consider surface tension or Coriolis force? Can viscosity be neglected (potential theory)?

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Introduction

Relevance of scale effects

Failure of Sines breakwater in 1978/9 which was strong enough in the scale model investigation (one reason for the failure were scale effects due to the incorrect scaling of the structural properties)

Sines breakwater failure, 1978/9

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Similarities

Perfect model-prototype similarity: Mechanical similarity

A physical scale model satisfying *mechanical similarity* is completely similar to its real-world prototype and involves no scale effects.

Mechanical similarity requires three criteria:

(i) **Geometric similarity**:
similarity in shape, i.e. all length dimensions in the model are λ times shorter than of its real-world prototype ($\lambda = L_P/L_M$)

(ii) **Kinematic similarity**:
geometric similarity and similarity of motion between model and prototype particles

(iii) **Dynamic similarity**:
requires geometric and kinematic similarity and in addition that all force ratios in the two systems are identical

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Similarities

Mechanical similarity (cont.)

Most relevant forces in fluid dynamics are:

Inertial force = mass × acceleration = $(\rho L^3)(V^2/L)$ = $\rho L^2 V^2$

Gravitational force = mass × gravitational acceleration = $\rho L^3 g$

Viscous force = dynamic viscosity × velocity/distance × area = $\nu (V/L) L^2$ = $\nu V L$

Surface tension force = unit surface tension × length = σL

Elastic compression force = Young's modulus × area = $E L^2$

Pressure force = unit pressure × area = $p L^2$

ρ (kg/m^3) = fluid density
L (m) = characteristic length
V (m/s) = char. velocity
p (N/m^2) = pressure
σ (N/m) = surface tensions
E (N/m^2) = Young's modulus
g (m/s^2) = gravitational acceleration
ν (kg/(ms)) = kinematic viscosity

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Similarities

Mechanical similarity (cont.)

Relevant force ratios are:

Froude number F = (inertial force/gravity force)$^{1/2}$ = $V/(gL)^{1/2}$

Reynolds number R = inertial force/viscous force = LV/ν

Weber number W = inertial force/surface tension force = $\rho L^2 V/\sigma$

Cauchy number C = inertial force/elastic force = $\rho V^2/E$

Euler number E = pressure force/inertial force = $p/\rho V^2$

Problem: Only the most relevant force ratio can be identical between model and its prototype, if identical fluid is used, and mechanical similarity is impossible. The most relevant force ratio is selected and the remaining result in *scale effects*.

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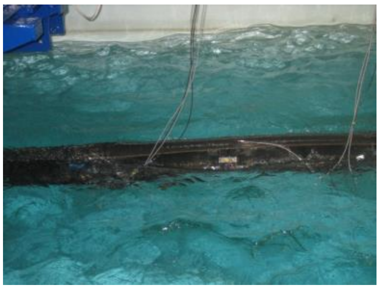
Similarities

Froude similarity $F_M = F_P$

For phenomena where gravity and inertial forces are dominant and effect of remaining forces such as kinematic viscosity are small.

Most water phenomena are modeled after Froude, in particular free surface flows (hydraulic structures, waves, wave energy converters etc.)

Anaconda WEC



Hydraulic jump modeled after Froude

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Similarities

Froude similarity $F_M = F_P$ (cont.)

How can Froude V_M be up-scaled?

$F_M = V_M/(g_M L_M)^{1/2} = V_P/(g_P L_P)^{1/2} = F_P$

with $g_M = g_P = g$ (not scaled)

$L_P = \lambda \cdot L_M$ (geometric similarity)

$F_M = V_M/(g L_M)^{1/2} = V_P/(g \lambda L_M)^{1/2} = F_P$

reduces to $V_M = V_P/\lambda^{1/2}$

$V_P = \lambda^{1/2} V_M$

Scale ratio $\lambda^{1/2}$ is required to upscale Froude model velocities

Scale ratios for Froude models

Parameter	Dimension	Froude	Reynolds
	Geometric similarity		
Length	[L]	λ	λ
Area	$[L^2]$	λ^2	λ^2
Volume	$[L^3]$	λ^3	λ^3
Rotation	[-]	1	1
	Kinematic similarity		
Time	[T]	$\lambda^{1/2}$	λ^2
Velocity	$[LT^{-1}]$	$\lambda^{1/2}$	λ^{-1}
Acceleration	$[LT^{-2}]$	1	λ^{-3}
Discharge	$[L^3T^{-1}]$	$\lambda^{5/2}$	λ
	Dynamic similarity		
Mass	[M]	λ^3	λ^3
Force	$[MLT^{-2}]$	λ^3	1
Pressure and stress	$[ML^{-1}T^{-2}]$	λ	λ^{-2}
Energy and work	$[ML^2L^{-2}]$	λ^4	λ
Power	$[ML^2T^{-3}]$	$\lambda^{7/2}$	λ^{-1}

Example: up-scaling model power P_M:

$\lambda = 20$, power model $P_M = 5$ Watts, power prototype P_P?

$P_P = \lambda^{7/2} P_M = 20^{7/2} 5 = 178885$ Watts = 0.18 MW!

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Similarities

Reynolds similarity $R_M = R_P$

For phenomena where viscous and inertial forces are dominant and effect of remaining forces such as gravity are small.

Not so often applied; examples include vortexes, tidal energy converters, sometimes rivers (water replaced by air to reach high model velocity)

Scale ratios for Reynolds models

Parameter	Dimension	Froude	Reynolds
Geometric similarity			
Length	[L]	λ	λ
Area	[L^2]	λ^2	λ^2
Volume	[L^3]	λ^3	λ^3
Rotation	[-]	1	1
Kinematic similarity			
Time	[T]	$\lambda^{1/2}$	λ^2
Velocity	[LT^{-1}]	$\lambda^{1/2}$	λ^{-1}
Acceleration	[LT^{-2}]	1	λ^{-3}
Discharge	[L^3T^{-1}]	$\lambda^{5/2}$	λ
Dynamic similarity			
Mass	[M]	λ^3	λ^3
Force	[MLT^{-2}]	λ^3	1
Pressure and stress	[$ML^{-1}T^{-2}$]	λ	λ^{-2}
Energy and work	[ML^2L^{-2}]	λ^4	λ
Power	[ML^2T^{-3}]	$\lambda^{7/2}$	λ^{-1}

Vortexes in river modeled with Reynolds

Example: up-scaling Reynolds model velocity v_M:

$\lambda = 20$, velocity model $v_M = 1$ m/s, velocity prototype v_P?

$v_P = \lambda^{-1} v_M = 1/20 = 0.05$ m/s => $v_M > v_P$!

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Scale effects

General

Scale effects are due to force ratios which are not identical between model and its prototype. Consequently, some forces are more dominant in the model than in the prototype and distort the results.

Four items are relevant, independent of a phenomenon:

(i) Physical hydraulic model tests with $\lambda \neq 1$ always involve scale effects. The relevant question is whether or not scale effects can be *neglected*.

(ii)The larger λ, the larger are scale effects. However, λ alone does not indicate whether or not scale effects can be neglected.

(iii) Each involved parameter requires its own judgement regarding scale effects. If e.g. wave height is not considerably affected by scale effects does not necessarily mean that e.g. air entrainment is also not affected (relative importance of forces may change).

(iv) Scale effects normally have a ‘damping’ effect. Parameters such as relative wave height or relative discharge are normally smaller in model than in its prototype. A judge-ment if prediction under or over-estimates prototype value is therefore often possible.

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Scale effects

General (cont.)

In a *Froude* model, scale effects are due to **R**, **W**, **C** and **E**.

In a *Reynolds* model, scale effects are due to **F**, **W**, **C** and **E**.

Scale effects due to **F** (in Reynolds models): reduced flow velocity (gravity)

Scale effects due to **R** (in Froude models): larger viscous losses in model, e.g. waves decay faster or energy dissipation is larger, water flows like honey

Scale effects due to **W** (in Froude and Reynolds models): too large air bubbles and faster air detrainment, wave celerity of short wave is affected, reduced discharge for small water depths

Scale effects due to **C** (in Froude and Reynolds models): structure (WEC) interacting with water behaves too stiff and strong (Sines break water), water and air are too hard in the model (impact phenomena, e.g. wave breaking)

Scale effects due to **E** (in Froude and Reynolds models): cavitation can not be observed in model if atmospheric pressure is not scaled (reduced)

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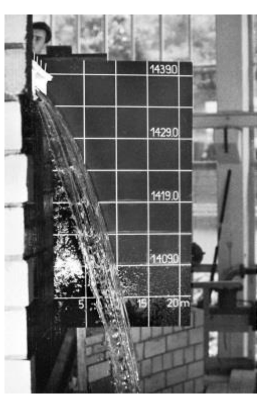
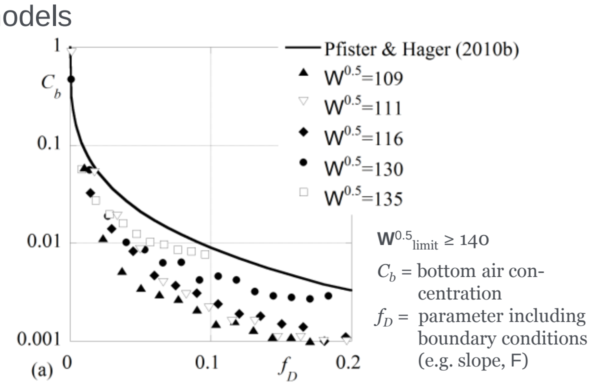
Scale effects

Examples: jet air entrainment and bottom air concentration on spillway in Froude models





Pfister and Chanson (2012)

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Reaching model-prototype similarity

4 available methods

Inspectional analysis: similarity *criteria* between model and prototype are found with set of equations describing a hydrodynamic phenomenon, which have to be identical between model and prototype.

Dimensional analysis: a method to transform dimensional in dimension-less parameters. Those *dimensionless* parameters have to be identical between model and prototype.

Calibration: calibration and validation of model tests with real-world data (discharge in river, run-up height of tsunami). The model is then applied with some confidence to other scenarios.

Scale series: a method comparing results of models of different sizes (different scale effects) to *quantify* scale effects.

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Reaching model-prototype similarity

Example: Landslide-tsunamis (Froude model)

Dimensional test parameters

Still water depth h
Slide impact velocity V_s
Bulk slide volume $\mathcal{V}_s$
Slide thickness s
Bulk slide density ρ_s
Slide impact angle α
Grain diameter d_g

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Reaching model-prototype similarity

Example: Dimensional analysis

A physical problem with n independent parameters q_1, ..., q_n can be reduced to a product of $n - r$ independent, dimensionless parameters Π_1, ..., Π_{n-r} with r as the minimum number of reference dimensions required to describe the dimensions of these n parameters. Each of Π_{n-r} have to be identical between model and prototype.

n = 9 independent par. q_n: h [L], V_s [LT^{-1}], $\forall_s$ [L^3], s [L], ρ_s [ML^{-3}], α [–], d_g [L], ρ [ML^{-3}], g [ML^{-2}]

r = 3 reference dimensions: [L], [T], [M]

$n - r$ = 6 dimensionless parameters: Π_1, ..., Π_6

r = 3 selected reference parameters: h, g, ρ (include different combinations of ref. dim.)

Example V_s: $\Pi_1 = V_s h^\beta g^\gamma \rho^\delta$ or $[-] = [LT^{-1}][L]^\beta[LT^{-2}]^\gamma[ML^{-3}]^\delta$

[L] : $0 = +1 + 1\beta + 1\gamma - 3\delta$

[T] : $0 = -1 + 0\beta - 2\gamma + 0\delta$

[M] : $0 = +0 + 0\beta + 0\gamma + 1\delta$ => $\beta = -1/2$, $\gamma = -1/2$ and $\delta = 0$

$\Pi_1 = V_s/(gh)^{1/2}$, the Froude number F

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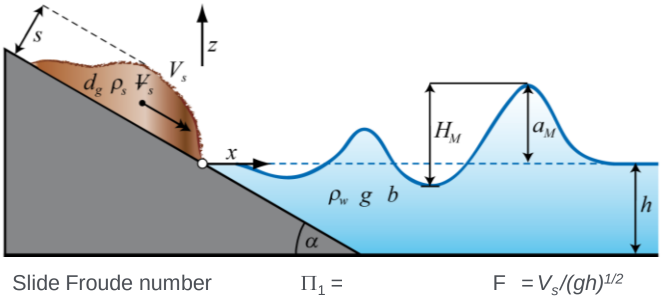
Reaching model-prototype similarity

Example: Dimensional analysis

All dimensionless parameters Π_1, ..., Π_6



Slide Froude number	Π_1 =	F $= V_s/(gh)^{1/2}$
Relative slide thickness	Π_2 =	$S = s/h$
Relative grain diameter	Π_3 =	$D_g = d_g/h$
Relative slide density	Π_4 =	$D = \rho_s/\rho_w$
Relative slide volume	Π_5 =	$V = \forall_s/(bh^2)$
Slide impact angle	Π_6 =	α

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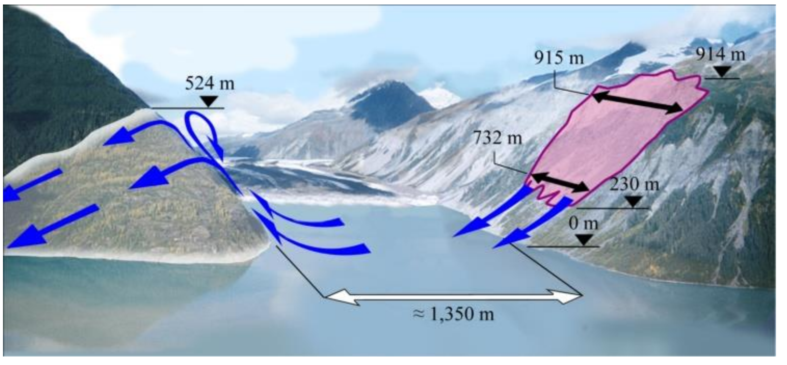
Reaching model-prototype similarity

Example: Calibration

Lituya Bay 1958 case



Run-up height observed in nature	R = 524 m
Run-up height measured in study of Fritz et al. (2001) at scale 1:675	R = 526 m

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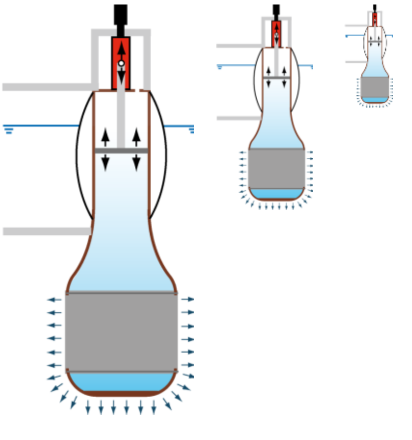
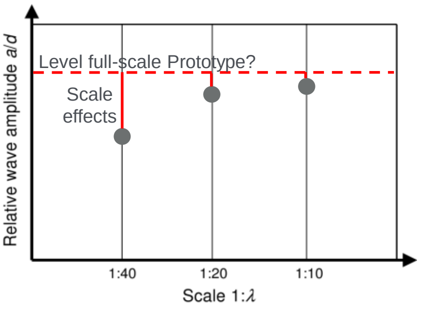
Reaching model-prototype similarity

Scale series: Results from tests conducted at three scales are compared



Quantification (schematic)



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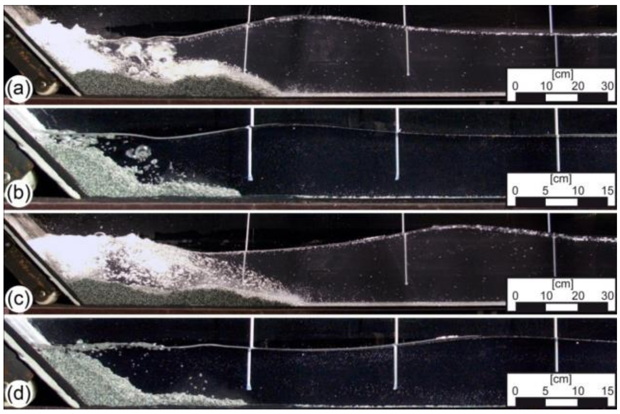
Reaching model-prototype similarity

Example: Scale series

Wave generation



F = 2.5
W = 5350
R = 290000

F = 2.5
W = 1340
R = 103400

F = 2.5
W = 5350
R = 290000

F = 2.5
W = 1340
R = 103400

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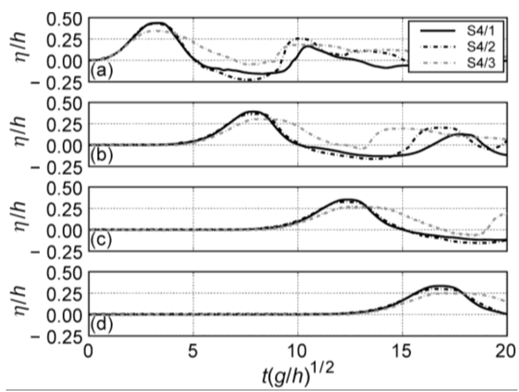
Reaching model-prototype similarity

Example: Scale series



S4/1: h = 0.400 m
S4/2: h = 0.200 m
S4/3: h = 0.100 m

Scale effects relative to a_M are negligible (<2%) if:

Reynolds number: R ≥ 300000 $R = g^{1/2}h^{3/2}/\nu_w$ char. velocity $V = (gh)^{1/2}$
Weber number: W ≥ 5000 $W = \rho_w g h^2/\sigma_w$ char. length $L = h$

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Dealing with scale effects

Avoidance: With rules of thumb

Satisfy limiting criteria
In Froude models: $R > R_{limit}$, $W > W_{limit}$ etc.

In practice rules of thumb are often applied. Some examples:

- Linear wave propagation is affected less than 1% by surface tension if $T > 0.35$ s (corresponding to $L > 0.17$ m, Hughes 1993)
- Free surface water flows should be > 5 cm to avoid significant surface tension scale effects (e.g. Heller et al. 2005)
- Wave height to measure wave force on slope during wave breaking should be larger than 0.50 m (Skladnev and Popov 1969)
- Free surface air-water flows should be conducted at $W^{0.5} > 140$ (Pfister and Chanson 2012)

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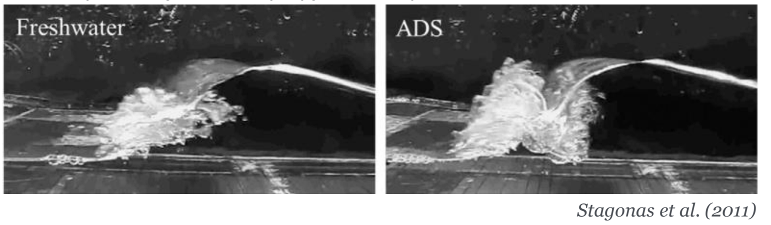
Dealing with scale effects

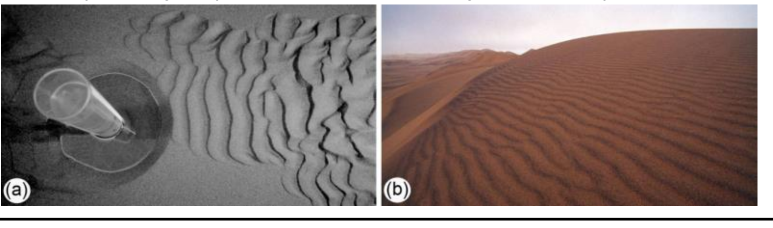
Avoidance: Replacement of fluid

Water replaced by water-isopropyl alcohol (reduced surface tension, increased W)



Stagonas et al. (2011)

Water replaced by air (reduced kinematic viscosity, increased R)



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Dealing with scale effects

Compensation

Compensation is achieved by distorting a model geometry by giving up exact geometric similarity of some parameters in favour of an improved model-prototype similarity.

Examples:

- Distorted models: the length λ_L scale factor of a model (say a river) is smaller than the height and width scale factor λ to compensate increased friction effects with a larger flow velocity
- The grain diameter d_g in sediment transport can often not be scaled with the same scale factor λ as the model main dimensions since it may result in $d_g < 0.22$ mm for which the flow-grain interaction characteristics changes. Zarn (1992) proposes a method to modify the model grain size distribution accordingly.

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Dealing with scale effects

Correction

Economic considerations, limited space or time may be reasons to intentionally build a small model where significant scale effects are expected. The model results may afterwards be corrected for phenomena where enough information on the quantitative influence of scale effects is available.

Examples:

- Solitary waves decay too fast in small scale physical models, which can be corrected with an analytical relation from Keulegan (1950)
- Correction factors for wave impact pressures from small-scale Froude models are included by Cuomo et al. (2010)
- Correction coefficients for the stability of rubble mound breakwater model tests were presented by Oumeraci (1984)

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Conclusions

- Similarity theory between physical model and real-world prototype was reviewed including mechanical, Froude and Reynolds similarities
- A model with $\lambda \neq 1$ always results in scale effects (with identical fluid) since only one relevant force ratio can be satisfied. The relevant question to ask is whether or not scale effects are negligible
- For each phenomenon or parameter in a model, the relative importance of the involved forces may vary and limitations should be defined relative to specific parameters and prototype features
- Inspectional analysis, dimensional analysis, calibration and scale series are available to obtain model-prototype similarity, to quantify scale effects, to investigate how they affect the parameters and to establish limiting criteria where they can be neglected
- Scale effects can be minimised with three methods namely avoidance, compensation and correction
- Similarity theory (scale effects) is not an exact science and requires engineering judgement for each particular problem

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